AI 539: TRUSTWORTHY ML
(CERTIFIED) DEFENSES AGAINST ADVERSARIAL EXAMPLES

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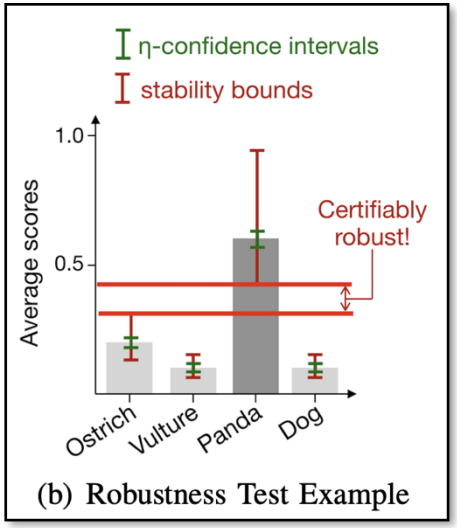
How can we make models "provably" robust?

Certified adversarial robustness via randomized smoothing, Cohen et al., ICML 2019

WHAT DOES IT MEAN BY "PROVABLY" ROBUST?

- Suppose:
  - (x, y): a test-time input and its oracle label
  - $x + \delta$: an adversarial example of x with small l_p–bounded (ε) perturbation δ
  - f: a neural network

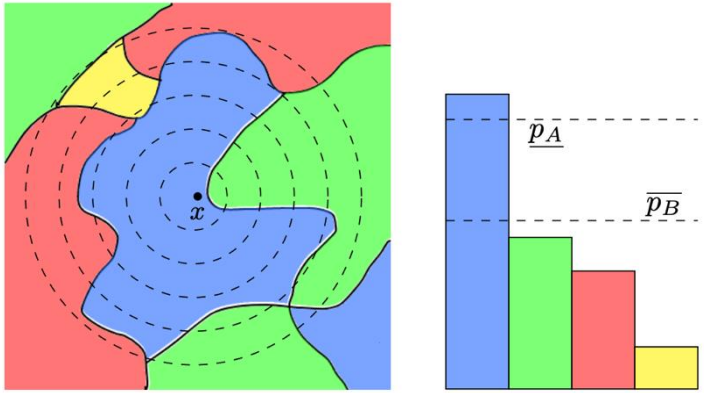
- Robustness:
  - For any δ where $||\delta||_p \leq \varepsilon$
  - The most probable class y_M for $f(x + \delta)$
  - Make f to be $\mathrm{P}[f(x + \delta) = y_M] > \max_{y \neq y_M} \mathrm{P}[f(x + \delta) = y]$



WHAT DOES IT MEAN BY "PROVABLY" ROBUST?

- Suppose:
  - (x, y): a test-time input and its oracle label
  - $x + \delta$: an adversarial example of x with small l_p–bounded (ε) perturbation δ
  - f: a neural network

- Robustness:
  - Most probable class: $\mathrm{P}[f(x+\delta) = c_A] \approx P_A$
  - A runner-up class : $\max_{y \neq y_M} \mathrm{P}[f(x+\delta) = y] \approx P_B$
  - "Provably" robust : $P_A > P_B$



How can you make your model provably robust?

- Randomized Smoothing:
  - Make a neural network f less sensitive to input details
  - Prior work:
    - Adversarial training (or robust training)
    - Denoising (we will talk about it in a bit later)

- Smoothing
  - In image processing: reducing noise (high frequency components)
  - In our context: reduce noise in inputs

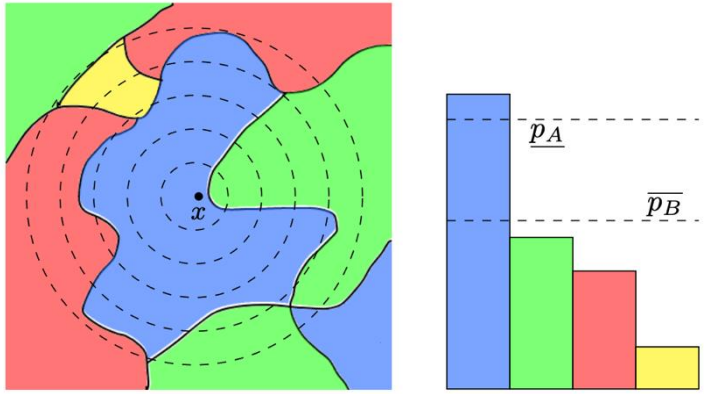
- Randomized
  - In statistics: the practice of using chance methods (random)
  - In this context: add Gaussian *random* noise to the input

How can you make your model provably robust?

- Certified robustness
  - Randomized smoothing transforms a base classifier f into a smoothed classifier g
  - The smoothed classifier g is robust around x with the l_2 radius of R

$$R = \frac{\sigma}{2}(\Phi^{-1}(\underline{p_A}) - \Phi^{-1}(\overline{p_B}))$$

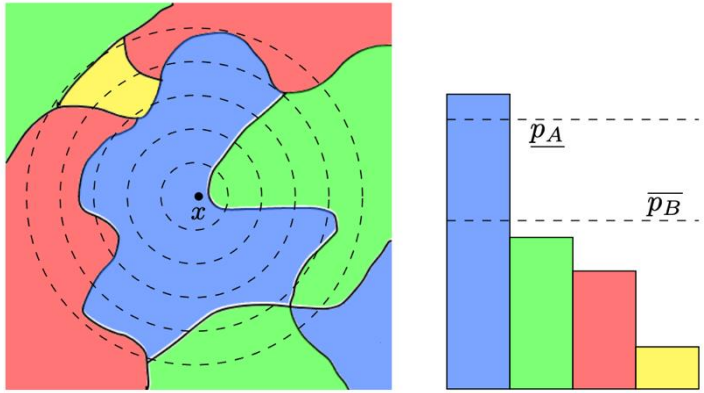
- Certification
  - g is a smoothed classifier
  - g outputs a prediction of c_A (a class)
  - within radius R around x
  - with a confidence of α



How can you make your model provably robust?

- Certification
  - g is a smoothed classifier
  - g outputs a prediction of c_A (a class)
  - within radius R around x
  - with a confidence of α

- Observations
  - R becomes large when we use high noise
  - R becomes infinite as $P_A \approx 1$ and $P_B \approx 0$



How can we certify the robustness?

- Practical algorithms for prediction and certification

Pseudocode for certification and prediction

 # evaluate g at x
function PREDICT(f, σ, x, n, α)
  counts $\leftarrow$ SAMPLEUNDERNOISE(f, x, n, σ)
  $\hat{c}_A, \hat{c}_B \leftarrow$ top two indices in counts
  $n_A, n_B \leftarrow$ counts$[\hat{c}_A]$, counts$[\hat{c}_B]$
  if BINOMPVALUE(n_A, $n_A + n_B$, 0.5) $\leq \alpha$ **return** $\hat{c}_A$
  else return ABSTAIN

 # certify the robustness of g around x
function CERTIFY(f, σ, x, n_0, n, α)
  counts0 $\leftarrow$ SAMPLEUNDERNOISE(f, x, n_0, σ)
  $\hat{c}_A \leftarrow$ top index in counts0
  counts $\leftarrow$ SAMPLEUNDERNOISE(f, x, n, σ)
  $\underline{p_A} \leftarrow$ LOWERCONFBOUND(counts$[\hat{c}_A]$, n, $1 - \alpha$)
  if $\underline{p_A} > \frac{1}{2}$ **return** prediction $\hat{c}_A$ and radius $\sigma\, \Phi^{-1}(\underline{p_A})$
  else return ABSTAIN

Guarantee the probability of $PREDICT$ returning a class other than $g(x)$ is α

How can we certify the robustness?

- Practical algorithms for prediction and certification

Pseudocode for certification and prediction

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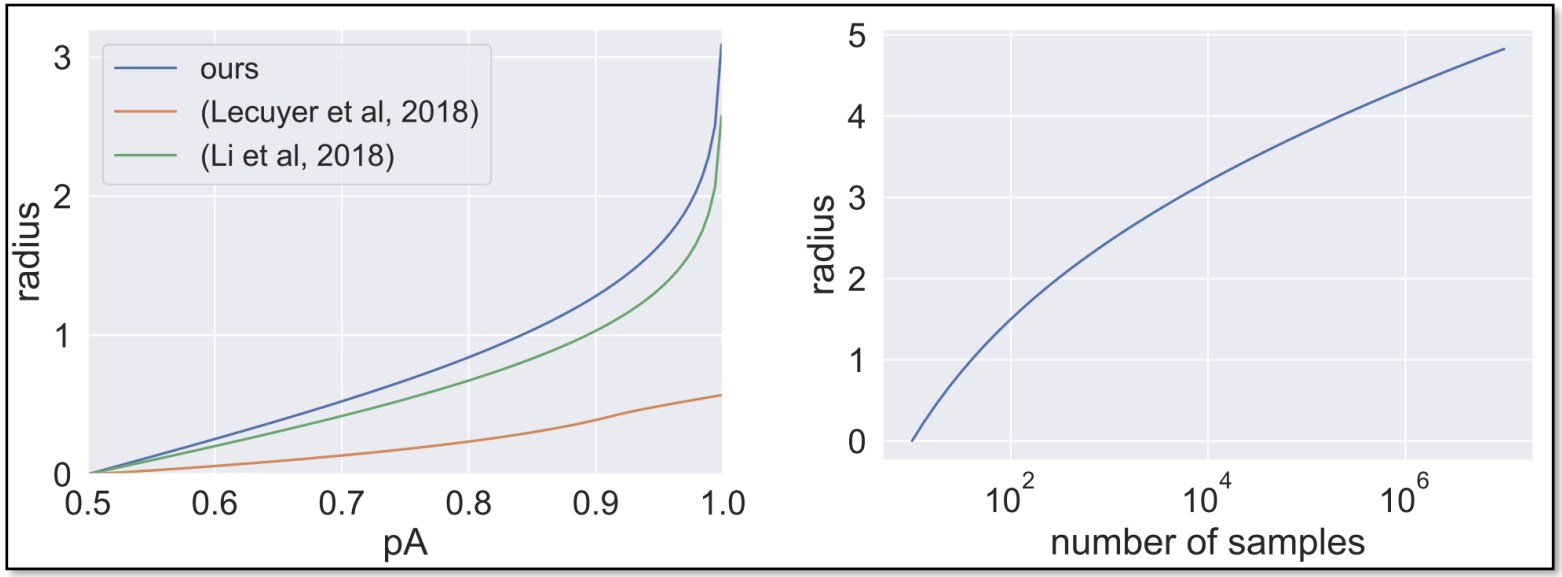
  else return ABSTAIN

Guarantee the probability of $PREDICT$ returning a class other than $g(x)$ is α

$CERTIFY$ returns a class c_A and a radius R for the $g(x)$ with the probability α

How can we certify the robustness?

- Practical algorithms for prediction and certification (empirical observation)
  - R becomes infinite as $P_A \approx 1$ and $P_B \approx 0$
  - The paper's algorithm offers a tighter estimation of R
  - The approximation of R becomes accurate if we use more samples



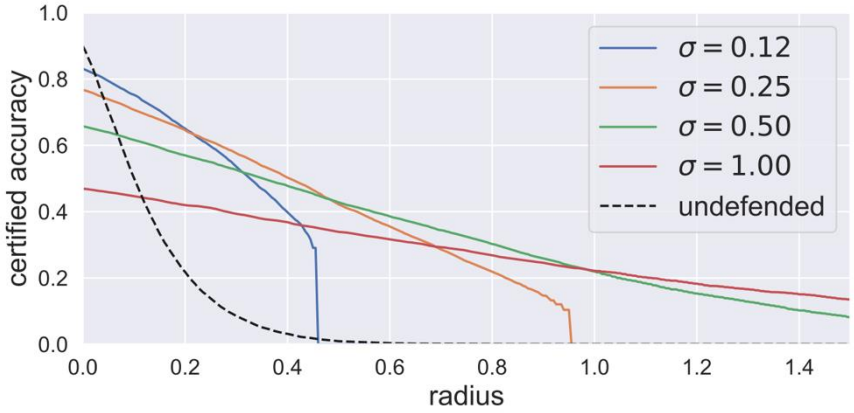
How can we certify the robustness?

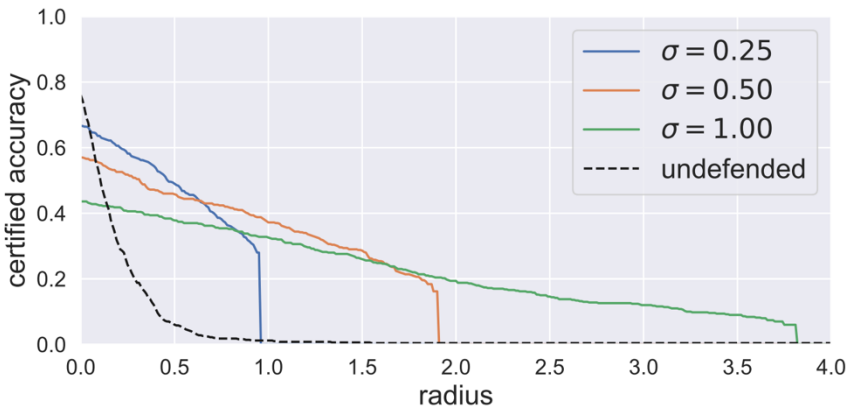
- Setup
  - CIFAR10: ResNet-110 and its full test-set
  - ImageNet: ResNet-50 and 500 random chosen test-set samples

- Measure
  - Certified test-set accuracy under a radius R with a confidence of α
  - Under various smoothing factor σ (std. of Gaussian noise used)

How can we certify the robustness?

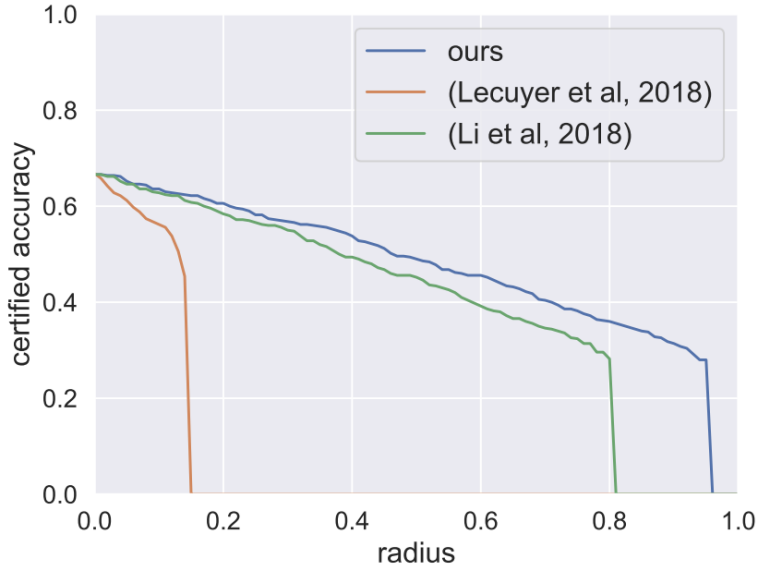
- Radius R vs. certified accuracy
  (left: CIFAR10, right: ImageNet)





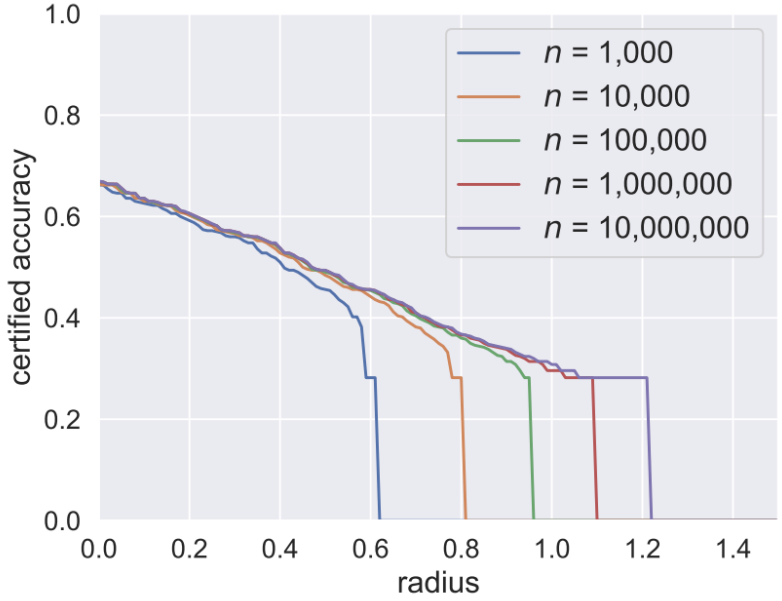
How can we certify the robustness?

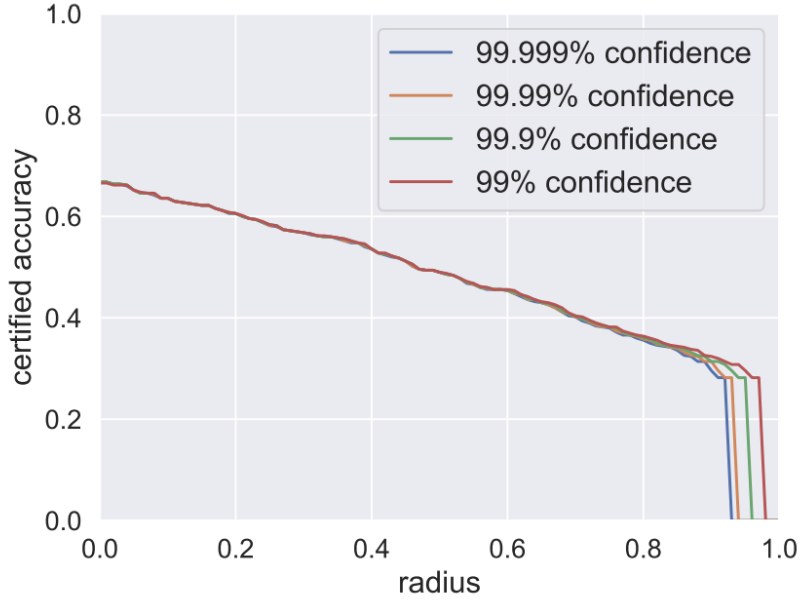
- Certified accuracy vs. prior work (ImageNet, $\sigma = 0.25$)



How can we certify the robustness?

- Certified accuracy vs. { # samples or confidence α }





"PROVABLY" ROBUST

- Research questions:
  - What does it mean by your model is robust?
    - A classifier f returns a prediction c within a radius R with a confidence α
  - How can you make your model provably robust?
    - Randomized smoothing (by Cohen et al.)
  - How can you certify that your model is robust?
    - Cohen et al., present practical algorithms for prediction and certification

How can we make certified defenses computationally feasible?

Denoised smoothing: a provable defense for pretrained classifiers, Salman et al., NeurIPs 2020

MAKING A SMOOTHED CLASSIFIER

- Conversion to a smoothed classifier g
  - Adversarial (or robust) training
  - Train a classifier f with noised samples $\sim N(x, \sigma^2 I)$ with x's oracle label

- Problem:
  - What if a classifier f is already trained?
  - Should we re-train all the classifiers, already on-service?

- Solution:
  - Denoised smoothing: train a denoiser that works with a pre-trained classifier

DENOISED SMOOTHING

- Conversion to a smoothed classifier
  - Train a denoiser $D_\theta: R^d \to R^d$ that removes the input perturbations for f
  - Pre-process an input x with the denoiser D_θ before x is fed to f
  - Pre-process step: generate noisy versions of x, denoise, and fed them to f

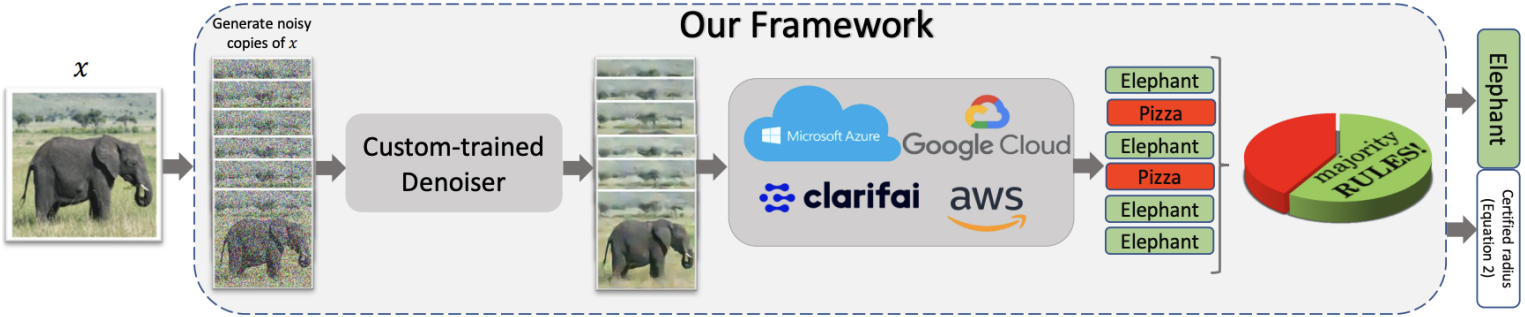


Figure 1: Given a clean image x, our denoised smoothing procedure creates a smoothed classifier by appending a denoiser to any pretrained classifier (e.g. online commercial APIs) so that the pipeline predicts in majority the correct class under Gaussian noise corrupted-copies of x. The resultant classifier is *certifiably* robust against ℓ_2-perturbations of its input.

DENOISED SMOOTHING

- Goal
  - Not to train f on noise
  - But, to provide certification to f

- Denoiser $D_\theta: R^d \rightarrow R^d$
  - $g(x) = \arg\max_{c \in \mathcal{Y}} \mathbb{P}[f(\mathcal{D}_\theta(x+\delta)) = c]$ where $\delta \sim \mathcal{N}(0, \sigma^2 I)$

- Training D_θ
  - **MSE** objective: Just train D_θ to remove Gaussian noise $L_{\text{MSE}} = \mathop{\mathbb{E}}_{\mathcal{S},\delta} \|\mathcal{D}_\theta(x_i+\delta) - x_i\|_2^2$
  - **+ Stability** objective: (White-box) Preserve f's predictions $L_{\text{Stab}} = \mathop{\mathbb{E}}_{\mathcal{S},\delta} \ell_{\text{CE}}(F(\mathcal{D}_\theta(x_i+\delta)), f(x_i))$

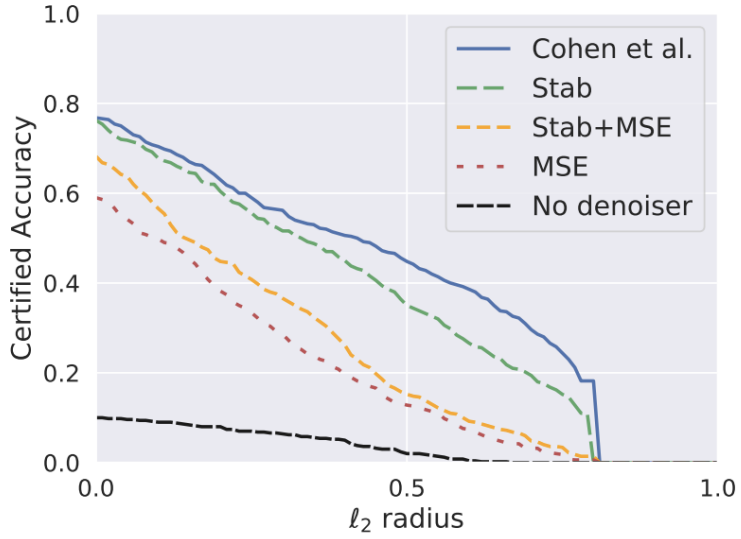
How can we certify the denoiser's robustness?

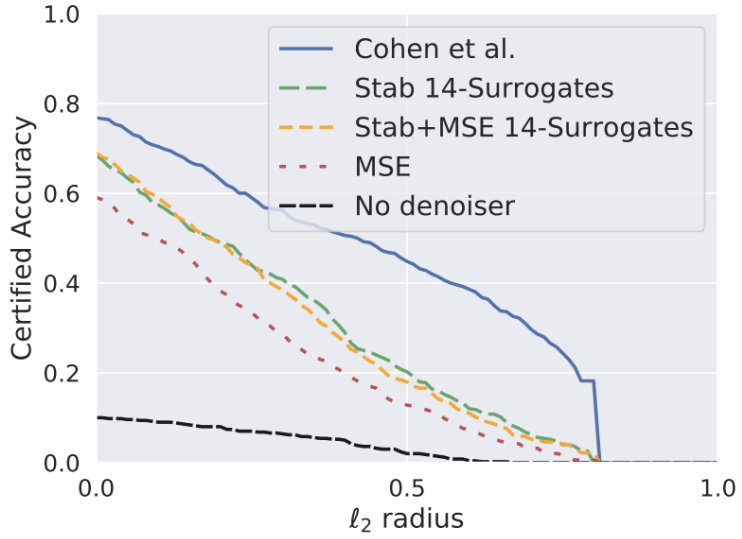
- Setup
  - ImageNet:
    - Pre-trained classifiers: ResNet-18/34/50 (white-box)
    - Baseline: ResNet-110 certified with $\sigma = 1.0$
  - Denoisers: DnCNN and MemNet trained with $\sigma = 0.25, 0.5, 1.0$
  - Objectives: MSE / Stab / Stab+MSE
    - White-box (as-is) | Black-box (14-surrogate models)

- Measure
  - Certified test-set accuracy under a radius R with a confidence of α
  - Under various smoothing factor σ (std. of Gaussian noise used)

How can we certify the denoiser's robustness?

- Certified accuracy vs. prior work (ImageNet, $\sigma = 0.25$)
  - (left: white-box) Denoiser offers certified accuracy close to that of Cohen et al.
  - (right: black-box) The certified accuracy is slightly smaller than the white-box case





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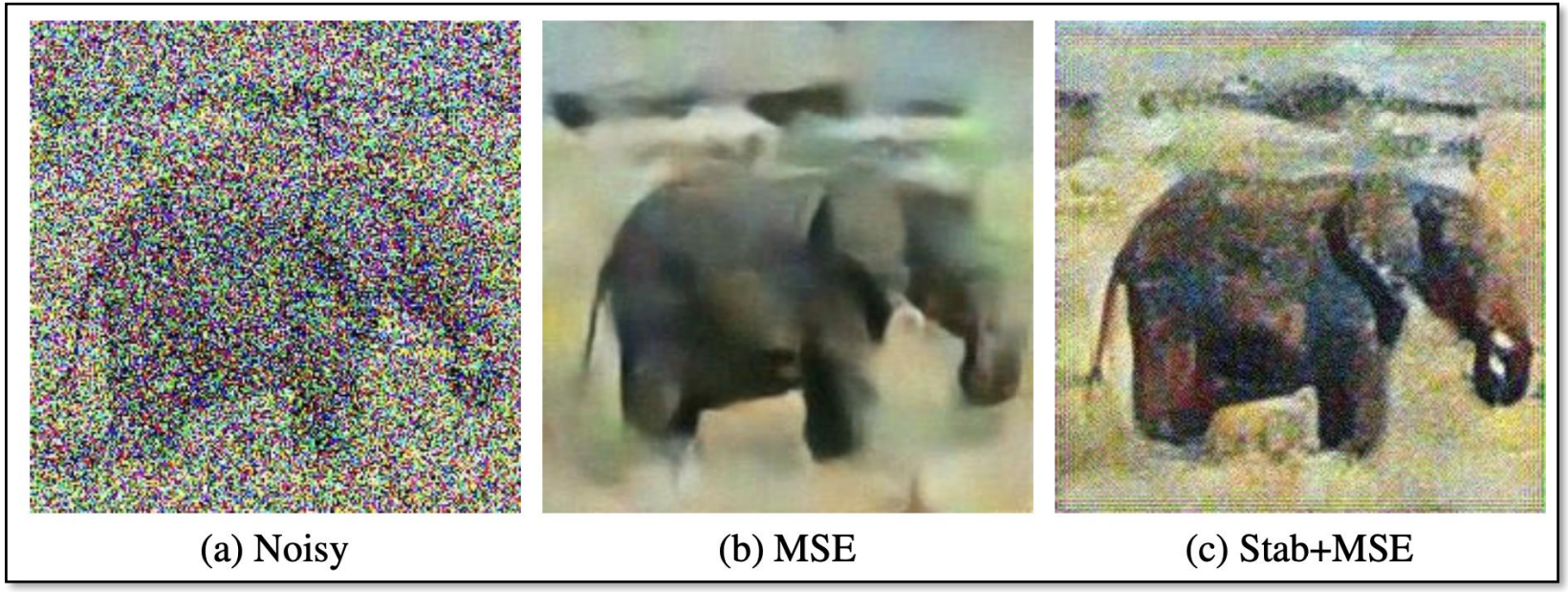
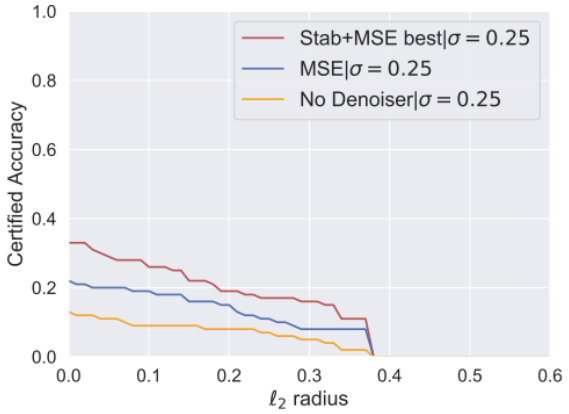


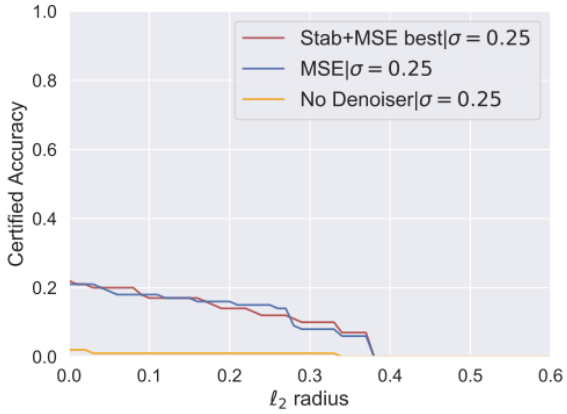
(a) Noisy (b) MSE (c) Stab+MSE

CAN WE CERTIFY OFF-THE-SHELF MODELS?

- Radius R vs. certified accuracy (with $\sigma = 0.25$)



(a) Azure



(b) Google Cloud Vision

CAN WE CERTIFY OFF-THE-SHELF MODELS?

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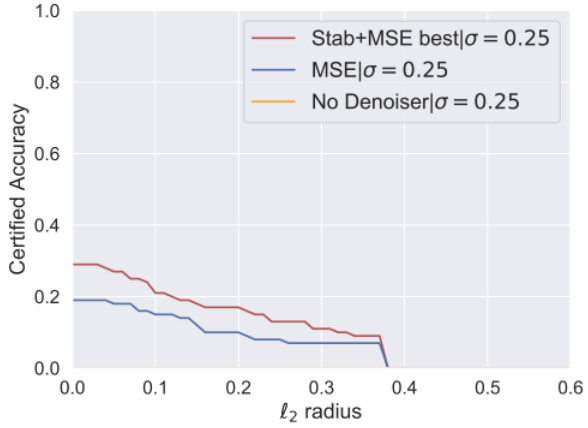
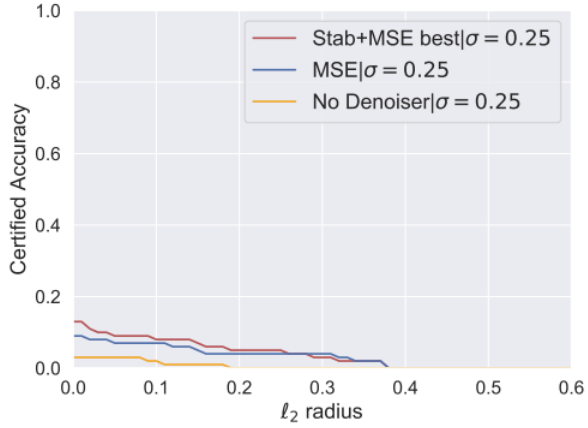


(c) Clarifai



(d) AWS

How can we get certified defenses for free?

(Certified!!) Adversarial robustness for free!, Calnini et al., ICLR 2023

DENOISED SMOOTHING: WHAT STILL NEEDS COMPUTATIONS?

- Goal
  - Not to train f on noise
  - But, to provide certification to f

- Denoiser $D_\theta: R^d \to R^d$
  - $g(x) = \arg\max_{c \in \mathcal{Y}} \mathbb{P}[f(\mathcal{D}_\theta(x+\delta)) = c]$ where $\delta \sim \mathcal{N}(0, \sigma^2 I)$

- Training D_θ
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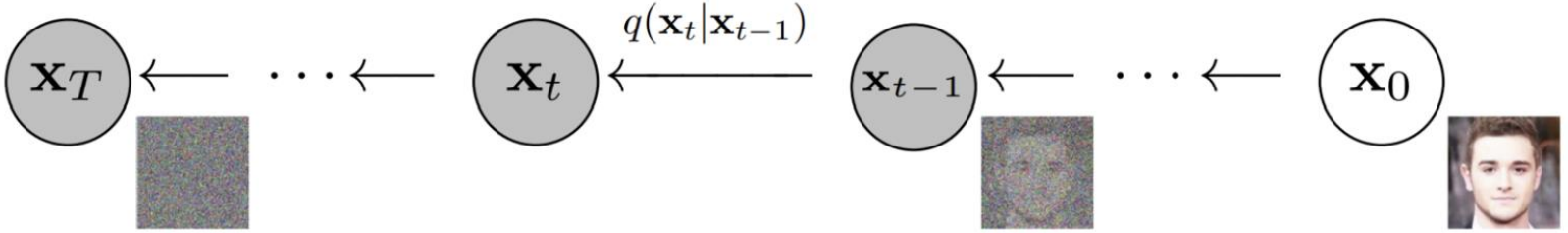
WE HAVE PRE-TRAINED DENOISERS

- Denoising diffusion probabilistic models (DDPMs)
  - Generative models trained to gradually denoise the data
  - The *diffusion* process transforms an image x to the purely random noise



  - Given an image x, the model samples a noisy image: $x_t \coloneqq \sqrt{\alpha_t} \cdot x + \sqrt{1-\alpha_t} \cdot \mathcal{N}(0, \mathbf{I})$
    α is a constant derived from t and determines the amount of noise to be added

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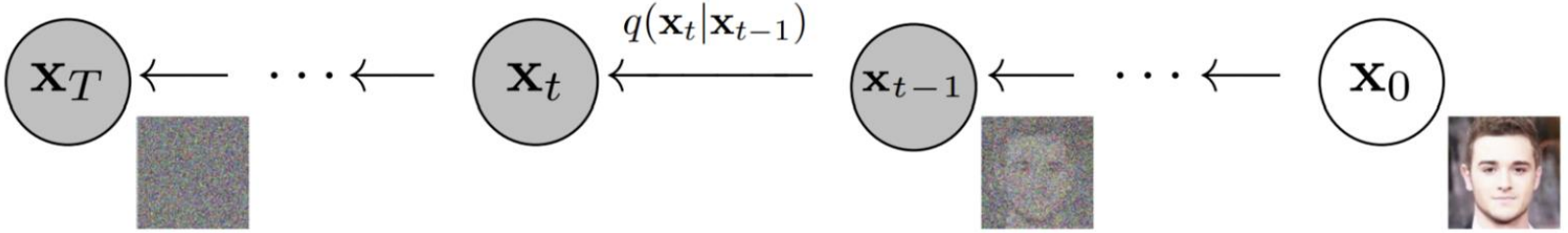
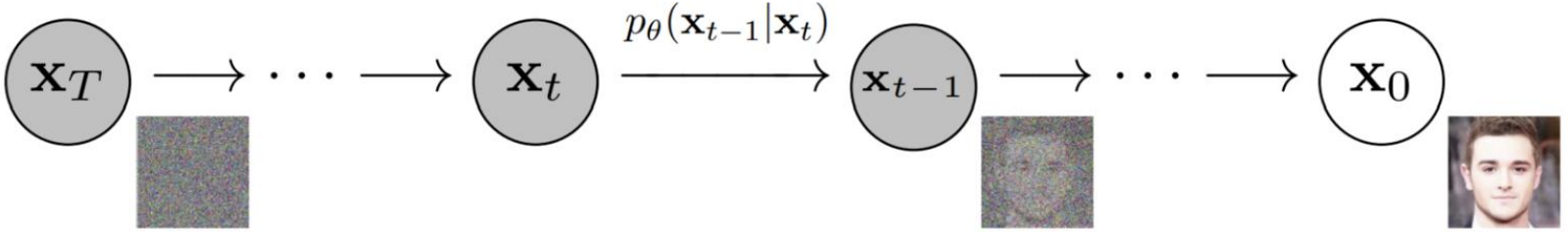


  - The *reverse* process synthesizes x from random Gaussian noise



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  - Generative models trained to gradually denoise the data
  - The *diffusion* process transforms an image x to the purely random noise
  - The *reverse* process synthesizes x from random Gaussian noise

- Use DDPMs as a denoiser $D_\theta: R^d \rightarrow R^d$
  - *One-shot* denoising: apply the diffusion model once for a fixed noise level
  - *Multi-step* denoising: apply the diffusion process multiple times

How can we certify the robustness?

- Practical algorithms for prediction and certification

Algorithm 2 Randomized smoothing (Cohen et al., 2019)

```
PREDICT(x, σ, N, η):
    counts ← 0
    for i ∈ {1, 2, ..., N} do
        y ← NOISEANDCLASSIFY(x, σ)
        counts[y] ← counts[y] + 1
    ŷ_A, ŷ_B ← top two labels in counts
    n_A, n_B ← counts[ŷ_A], counts[ŷ_B]
    if BINOMPTEST(n_A, n_A + n_B, 1/2) ≤ η then
        return ŷ_A
    else
        return Abstain
```

Guarantee the probability of $PREDICT$ returning a class other than $g(x)$ is α

Algorithm 1 Noise, denoise, classify

```
NOISEANDCLASSIFY(x, σ):
    t*, α_t* ← GETTIMESTEP(σ)
    x_t* ← √α_t*(x + N(0, σ²I))
    x̂ ← denoise(x_t*; t*)
    y ← f_clf(x̂)
    return y

GETTIMESTEP(σ):
    t* ← find t s.t. (1 − α_t)/α_t = σ²
    return t*, α_t*
```

How can we certify the denoiser's robustness?

- Setup
  - Data: CIFAR-10 and ImageNet-21k
  - Model: Wide-ResNet-28-10 (white-box)
  - Denoisers: DDPMs

- Measure
  - Certified test-set accuracy under a radius R with a confidence of α
  - Under various smoothing factor ε (std. of Gaussian noise used)

How can we certify the denoiser's robustness?

- Certified accuracy vs. prior work (ImageNet-21k)
  - DDPM denoisers offer the highest certified accuracy compared to the prior work
  - To achieve the highest accuracy, one can use this off-the-shelf model w/o training

			Certified Accuracy at ε (%)				
Method	Off-the-shelf	Extra data	0.5	1.0	1.5	2.0	3.0
PixelDP (Lecuyer et al., 2019)	○	✗	$^{(33.0)}$16.0	-	-		
RS (Cohen et al., 2019)	○	✗	$^{(67.0)}$49.0	$^{(57.0)}$37.0	$^{(57.0)}$29.0	$^{(44.0)}$19.0	$^{(44.0)}$12.0
SmoothAdv (Salman et al., 2019)	○	✗	$^{(65.0)}$56.0	$^{(54.0)}$43.0	$^{(54.0)}$37.0	$^{(40.0)}$27.0	$^{(40.0)}$20.0
Consistency (Jeong & Shin, 2020)	○	✗	$^{(55.0)}$50.0	$^{(55.0)}$44.0	$^{(55.0)}$34.0	$^{(41.0)}$24.0	$^{(41.0)}$17.0
MACER (Zhai et al., 2020)	○	✗	$^{(68.0)}$57.0	$^{(64.0)}$43.0	$^{(64.0)}$31.0	$^{(48.0)}$25.0	$^{(48.0)}$14.0
Boosting (Horváth et al., 2022a)	○	✗	$^{(65.6)}$57.0	$^{(57.0)}$44.6	$^{(57.0)}$**38.4**	$^{(44.6)}$**28.6**	$^{(38.6)}$**21.2**
DRT (Yang et al., 2021)	○	✗	$^{(52.2)}$46.8	$^{(55.2)}$44.4	$^{(49.8)}$**39.8**	$^{(49.8)}$**30.4**	$^{(49.8)}$**23.4**
SmoothMix (Jeong et al., 2021)	○	✗	$^{(55.0)}$50.0	$^{(55.0)}$43.0	$^{(55.0)}$**38.0**	$^{(40.0)}$26.0	$^{(40.0)}$20.0
ACES (Horváth et al., 2022b)	◐	✗	$^{(63.8)}$54.0	$^{(57.2)}$42.2	$^{(55.6)}$35.6	$^{(39.8)}$25.6	$^{(44.0)}$19.8
Denoised (Salman et al., 2020)	◐	✗	$^{(60.0)}$33.0	$^{(38.0)}$14.0	$^{(38.0)}$6.0	-	-
Lee (Lee, 2021)	●	✗	41.0	24.0	11.0	-	-
Ours	●	✓	$^{(82.8)}$**71.1**	$^{(77.1)}$**54.3**	$^{(77.1)}$**38.1**	$^{(60.0)}$**29.5**	$^{(60.0)}$13.1

How can we certify the denoiser's robustness?

- One-shot vs. multi-step denoising (ImageNet-21k)
  - One-shot denoising offers more faithful results
  - Multi-step denoising destroys the information about the original image

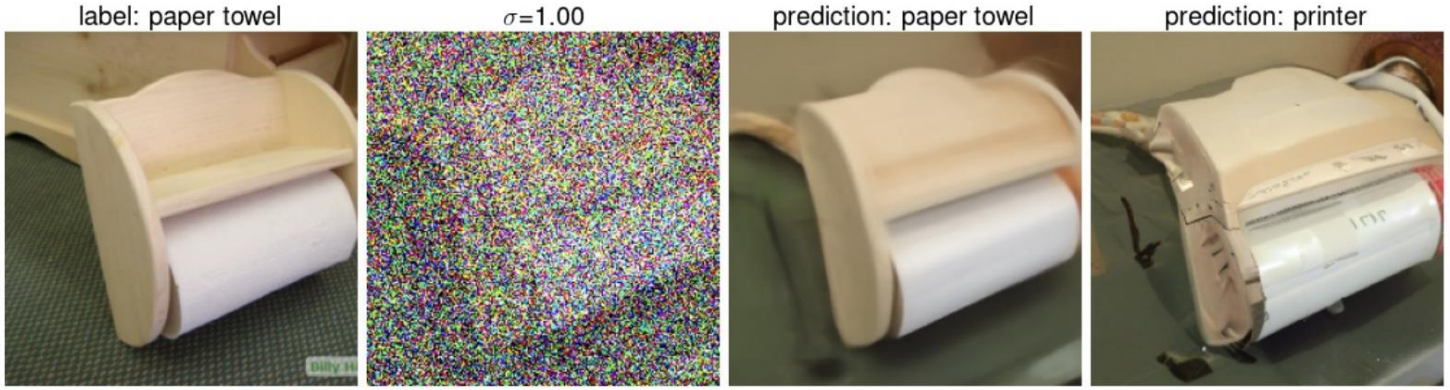

Figure 3: Intuitive examples for why multi-step denoised images are less recognized by the classifier. From left to right: clean images, noisy images with $\sigma = 1.0$, one-step denoised images, multi-step denoised images. For the denoised images, we show the prediction by the pretrained BEiT model.

OTHER WORK ON THE "PROVABLE" ROBUSTNESS

- Further readings
  - PixelDP (Lecuyer *et al.*): Use differential privacy (DP) for the certification
  - Li *et al.*: Propose a tighter bound for the certification, based on Renyi-divergence

Thank You!

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https://secure-ai.systems/courses/MLSec/F23